OMC

Ontario Invitational Mathematics Examination

Saturday, November 23, 2024

Contestant Information:

First name

Contestant ID

O						

Last name

General Instructions:

1. **DO NOT** open the contest booklet until instructed by your proctor.
2. Diagrams are **NOT** drawn to scale. They are intended as aids only.
3. You are allowed scratch paper, a ruler, a compass, and a protractor for rough work.
4. Calculators are not permitted during this contest.
5. Express answers as simplified exact numbers as a sum of products unless otherwise indicated. For example, $\pi + 1$ and $1 + \sqrt{2}$ are simplified exact numbers.
6. If the space we provide is not sufficient for you to present your solution, you may use additional blank sheets and label them with your name, contestant ID, and question number.

Exam Format:

1. The OIME consists of two parts Part A and B to be completed in 150 minutes.
   Part A
   1. This part consists of seven questions, each worth 10 marks.
   2. A correct answer is worth full marks, but partial marks may be given only if **relevant** work is shown in the space provided.

   Part B
   1. This part consists of three questions, each worth 10 marks.
   2. Full marks are awarded for a correct answer and clear, complete solutions written in the appropriate location in the answer booklet.

Problem 1

Let α be the number that is created by concatenating the first 26 positive integers, $\alpha = 1234567\ldots242526$. What is the remainder when α is divided by 99? (**Short Answer**)

Your final answer:

Problem 2

10 students, labeled P_0, P_1, to P_9 are on their way to a math contest when one of them robs a bank. They are now being interrogated and make up two statements. Every student P_i says, "P_{i+1} is a liar" and "P_{i+2} or P_{i+3} robbed the bank" (taking indices modulo 10, so $P_{10} = P_0$). How many of the 20 statements are true? (**Short Answer**)

Your final answer:

Problem 3

Shanna and Elaine are in the airport, and they are standing at the left end of a 108m moving walkway, which is moving right at a speed of 2m/s. Shanna walks beside the moving walkway on the regular ground (the ground travels at 0m/s) at a constant speed. Elaine is very energetic, and her speed on the regular ground is 4m/s. Elaine runs on the moving walkway until she reaches the right end, then turns back and runs on the moving walkway until she meets Shanna. Then she immediately turns back and repeats this. After doing this twice, she runs for $\frac{50}{9}$ seconds, reaching the right end of the walkway where she waits for Shanna. Find how fast Shanna walks, in m/s. (**Short Answer**)

Your final answer:

Problem 4

Let ellipse C be defined by the equation

$$\frac{x^2}{25} + \frac{y^2}{27} = 1.$$

Suppose $\triangle ABC$ is a triangle that completely covers the ellipse. Determine the minimum value of the area of $\triangle ABC$. (**Short Answer**)

Your final answer:

Problem 5

Square S has vertices at $(0,0),(6,0),(0,6)$, and $(6,6)$. If two lattice points are chosen at random in the interior of S, what is the probability that the line passing through the two points divides S into two regions with equal area? (**Short Answer**)

Your final answer:

Problem 6

Let E be a point in unit square $ABCD$ such that $AE = 1$. Let M be the midpoint of BC. What is the distance of E to AD when $\angle CEM$ is maximized? (**Short Answer**)

Your final answer:

Problem 7

Let $a_1, a_2, \ldots a_{119}$ be a strictly increasing arithmetic sequence of positive integers with common difference d and $a_1 = 119$. Find the number of possible values $d \leq 119$ so that there exist positive integers $1 < i < j \leq 119$ such that $119, a_i, a_j$ forms a geometric sequence. (**Short Answer**)

Your final answer:

Problem 8

$P(x)$ is a polynomial with both positive and negative coefficients. If $P(x)^2$ has only positive or zero coefficients, what is the minimum possible degree of $P(x)$? (**Full Solution**)

(Question 8 cont'd)

Problem 9

K is an infinite sequence of positive integers such that the sum of any number of consecutive terms in K does not equal to 2025. Determine, with proof, the general expression for the minimum sum of the first n terms in sequence K. (**Full Solution**)

(Question 9 cont'd)

Problem 10

Prove that there are infinitely many triples of positive integers (a, b, c) that satisfy the equation $a^2 + b^2 = 2c^2 + 1$. (**Full Solution**)

(Question 10 cont'd)

USE THIS PAGE IF ADDITIONAL SPACE IS REQUIRED

Clearly state the question number being answered and refer the marker to this page.