Tesseract

# *Tesseract Mathematics Challenge*

*Monday, March 4, 2024 -*
*Friday, March 8, 2024*

**Contestant Information:**

First name

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Last name

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Contestant ID

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**General Instructions:**

1. **DO NOT** open the contest booklet until instructed by your proctor.
2. **DO NOT** discuss the problems or solutions from this contest until after 11:59 pm, Sunday, March 10, 2024.
3. You are allowed scratch paper, a ruler, a compass, and a protractor for rough work.
4. Express answers as simplified exact numbers or as a sum of products unless otherwise indicated. For example, $\pi + 1$ and $1 + \sqrt{2}$ are simplified exact numbers.
5. If the space we provide is not sufficient for you to present your solution, you may use additional blank sheets and label them with your name, contestant ID, and question number.
6. Calculators are permitted as long as they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software. Graphing calculators (GDCs) are NOT allowed.

**Exam Format:**

- The TMC consists of five two-part problems, worth 10 marks each, to be completed in 60 minutes.

  **Part A (Short Answer)**
  - A correct answer is worth 3 marks, but partial marks may be given only if **relevant** work is shown in the space provided.

  **Part B (Full Solution)**
  - 7 marks are awarded for a correct answer and clear, complete solutions written in the appropriate location in the answer booklet.

# Problem 1

A person's birth score is given by multiplying the month and date of their birthday together. For example, a person born on February 9th would have a birth score of $2 \times 9 = 18$.

(a) What is the birth score of a person born on December 25?

**Your final answer:**

(b) Naomi and Carl both have a birth score of 252, with Naomi's birthday being before Carl's. How many days later is Carl's birthday compared to Naomi's?

# Problem 2

Ms. Yang bought 500 candies from the supermarket using one dollar, five dollar, and ten dollar bills, with one candy costing exactly 1 dollar.

(a) If she uses exactly three 10 dollar bills but any number of 5 dollar and 1 dollar bills, how many ways can Ms. Yang pay for the candy?

**Your final answer:**

(b) How many possible ways can she pay for the candy if she can use any number of each of the bills?

# Problem 3

Tommy and Freddy are learning about function transformations in class. Getting extremely bored, Tommy decided to doodle functions on his notebook. He doodles the graph of $y = f(x)$ on one page, and doodles another graph of $y = g(x)$ on another page by reflecting $f(x)$ across the y-axis and translating it to the right by 4 units. Interestingly, he finds that $f(x) + g(x) = 10$ for all real numbers $x$.

(a) Tommy discovered that by turning $f(x)$ about point $(m, n)$ by 180 degrees, it turns back into $f(x)$ again! What is $m + n$? (This point works for all possible functions $f(x)$)

**Your final answer:**

(b) Freddy, being the excellent student that he is, listened carefully in class (be like Freddy!). He grabbed Tommy's page with the graph $y = f(x)$ on it and started practicing. He drew the graph $y = 2024(3x - 6)^{2025} + 5$ on top of $y = f(x)$. If there are exactly 2023 intersections between the two graphs, what is the sum of all $x$-coordinates and $y$-coordinates of those intersection points?

# Problem 4

Let $ABC$ be a triangle with $AB = 13, BC = 14, AC = 15$. Let $D$ be on $BC$ such that $AD$ bisects $\angle BAC$, and $E$ on $AC$ with $DE \perp AC$.

(a) Find the length of $AD$.

**Your final answer:**

(b) Extend $ED$ past $D$ to intersect line $AB$ at $X$. Find the length of $BX$.

# Problem 5

Shiro is playing a game where she has a score of $x$ which is initially equal to 1. For each positive integer $2, 3, \ldots, 100$ (in that order), Shiro can choose one of the following actions:

- Add the number to $x$.
- Divide $x$ by the number.

However, Shiro must never let $x$ become less than 1 after any action.

(a) Compute the minimum possible value of $x$ after using all 99 numbers.

**Your final answer:**

(b) Find the number of possible values of $\lfloor x \rfloor$ after using all 99 numbers.
The floor function, $\lfloor x \rfloor$, takes a real number $x$, and gives as output the greatest integer less than or equal to $x$. For example, $\lfloor 2.1 \rfloor = 2$ and $\lfloor -2.6 \rfloor = -3$

(b) continued