Tesseract Mathematics Challenge

Monday, March 3, 2025 -
Friday, March 7, 2025

Contestant Information:

Contestant ID

	-						

Contest Serial Number:

General Instructions:

1. **DO NOT** open the contest booklet until instructed by your proctor.
2. **DO NOT** discuss the problems or solutions from this contest until after 11:59 pm, Sunday, March 9, 2025.
3. You are allowed scratch paper, a ruler, a compass, and a protractor for rough work.
4. Express answers as simplified exact numbers unless otherwise indicated. For example, $\pi + 1$ and $1 + \sqrt{2}$ are simplified exact numbers.
5. If the space we provide is not sufficient for you to present your solution, you may use additional blank sheets and label them with your contestant ID, and question number.
6. Calculators are permitted as long as they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.
7. Graphing calculators (GDCs) are NOT allowed.

Exam Format:

- The TMC consists of five two-part problems, worth 10 marks each, to be completed in 60 minutes.
  Short Answer
  - A correct answer is worth 2 to 3 marks as indicated, but partial marks may awarded if **relevant** work is shown in the space provided.

  Full Solution
  - The remaining marks are awarded for a correct answer and clear, complete solutions written in the appropriate location in the answer booklet.

Problem 1

Emily has a bag which contains 3 red balls, 2 blue balls, 5 green balls and no balls of any other colour.

(a) If she randomly removes a ball from the bag, what is the probability that it is blue?
(**Short Answer**; 3 marks)

Your final answer:

(b) She adds another n red balls to the bag, and now the probability that a randomly removed ball is red is $\frac{1}{2}$. What is the value of n? (**Full Solution**; 7 marks)

Problem 2

(a) (i) Find the unique positive real number x such that $x, \frac{x}{2}, x+1$ forms an arithmetic sequence in that order. (**Short Answer**; 2 marks)

Your final answer:

(ii) Find the unique positive real number x such that $x, 3x+1, x^2$ forms an arithmetic sequence in that order. (**Short Answer**; 3 marks)

Your final answer:

(b) Find the unique positive real number x such that x, $\lfloor x \rfloor$, and $x - \lfloor x \rfloor$ forms a arithmetic sequence in that order. (The floor function $\lfloor x \rfloor$ evaluates to the largest integer less than or equal to x)
(**Full Solution**; 5 marks)

Problem 3

For a positive integer n, define $S(n)$ to be the sum of the digits of the base-10 representation of n. For example, $S(6150) = 6 + 1 + 5 + 0 = 12$.

(a) Compute the number of two digit positive integers n such that $S(n) = S(2n)$.
(**Short Answer**; 3 marks)

Your final answer:

(b) Find the smallest positive integer n such that $S(n)$ and $S(n + 11)$ are both divisible by 13.
(**Full Solution**; 7 marks)

Problem 4

A game called "24 point" is played by giving the player four positive integers and asking them to make the number 24 by only using addition, subtraction, multiplication, division, and parenthesis. For example, the winning operation for 4, 4, 10, 10 is (10x10-4)/4 = 24. Note that there may be multiple winning operations for a given set of four positive integers.

(a) Determine a winning operation for 2, 5, 5, 10. (**Short Answer**; 2 marks)

Your final answer:

(b) Given any four positive integers, prove that by only using addition, subtraction, multiplication, division, and parenthesis, a multiple of 24 can be always be made. (**Full Solution**; 8 marks)

(b) continued

Problem 5

In non-isosceles acute triangle ABC, I is the incenter. The angle bisector of B intersects the altitude from A at D. The angle bisector of C intersect the altitude from A to E. The circumcircle of $\triangle AID$ intersects line CE at $X \neq I$. The circumcircle of $\triangle AIE$ intersects line BD at $Y \neq I$.
(If you define any additional points other than the ones given in the problem please include a labelled diagram, there will be a 1 mark deduction otherwise)

(a) Prove that XY is parallel to BC. (**Full Solution**; 4 marks)

(b) Let P be the point such that $PB \parallel AY$ and $PC \parallel AX$. Prove that P is on the circumcircle of $\triangle ABC$. (**Full Solution**; 6 marks)

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