1. If $p = 7$ and $q = 2(p - 5)$, compute $q(p + 5) - 2$.

(A) 286 **(B)** 142 **(C)** 100 **(D)** 61 **(E)** 46

proposed by: Aidan Zhang

Solution: Substituting $p = 7$ into the equation for q yields $q = 2(7 - 5) = 2(2) = 4$. Then, $q(p + 5) - 2 = 4(7 + 5) - 2 = 4(12) - 2 = \boxed{\textbf{(E) } 46}$.

2. The operation § is defined by $x\S y = x^2 + xy + y^2$. Find $(1\S 2)\S 3$.

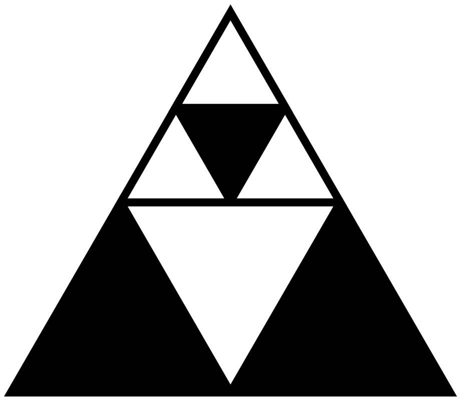
(A) 49 **(B)** 58 **(C)** 70 **(D)** 79 **(E)** 91

proposed by: Aidan Zhang

Solution:

$$\begin{aligned}(1\S 2)\S 3 &= (1^2 + 1 \cdot 2 + 2^2)\S 3 \\ &= 7\S 3 \\ &= 7^2 + 7 \cdot 3 + 3^2 \\ &= \boxed{\textbf{(D) } 79}\end{aligned}$$

3. In the figure shown, an equilateral triangle is split into four congruent smaller triangles, one of which is split again in the same way. What fraction of the figure is shaded?



(A) $\frac{1}{2}$ **(B)** $\frac{13}{24}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{9}{16}$ **(E)** $\frac{7}{12}$

proposed by: Aidan Zhang

Solution: In total, there are $4 \cdot 4 = 16$ equilateral triangles of the smallest size in the figure. Of these, 9 are shaded, so the total shaded area is $\boxed{\textbf{(D) } \frac{9}{16}}$.

4. Compute $2021 + \dfrac{2023! + 2024!}{2022! \times 2025}$, where $n! = 1 \times 2 \times \ldots \times (n - 1) \times n$.

(A) 4044 **(B)** $\frac{4044}{2023}$ **(C)** $\frac{4088484}{2023}$ **(D)** $\frac{4094552}{2023}$ **(E)** 2023

proposed by: Aidan Zhang

Solution: First, examine the expression $\dfrac{2023! + 2024!}{2022! \times 2025}$. Since $2023! = 2022! \times 2023$ and $2024! = 2022! \times 2023 \times 2024$, the numerator can be factored as $2022!(2023 + 2023 \times 2024)$. Now, 2023 can be factored out of $2023 + 2023 \times 2024$ to yield $2023(1 + 2024) = 2023 \times 2025$. The numerator is therefore $2022! \times 2025 \times 2023$. Cancelling with the $2022! \times 2025$, the expression ends up as just 2023.

Finally, 2021 is added for the final answer of $\boxed{\textbf{(A) } 4044}$.

5. Cards are drawn one at a time from a deck containing 3 red cards and 2 black cards. What is the probability that all the black cards are drawn before all the red cards are drawn?

(A) $\frac{2}{5}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{7}{10}$ **(D)** $\frac{3}{5}$ **(E)** $\frac{2}{3}$

proposed by: Daniel Chen

Solution: We are looking for the probability that the last card drawn is not black. This is simply the probability that the last card is red, which is $\boxed{\textbf{(D)}\ \frac{3}{5}}$.

6. Positive numbers x, y, a, b satisfy the inequalities $x < y$ and $a < b$. How many of the following inequalities are necessarily true?

(i) $x + a - b < y - a + b$

(ii) $\frac{x+y}{b} < \frac{x+y}{a}$

(iii) $\frac{x}{b} < \frac{y}{a}$

(iv) $a - y < b - x$

(v) $ax < by$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

proposed by: Aidan Zhang

Solution: All $\boxed{\textbf{(E)}\ 5}$ inequalities are true.

(i) $x + a - b < y - a + b \iff x + 2a < y + 2b$, which is true because $x < y$ and $2a < 2b$.

(ii) $\frac{x+y}{b} < \frac{x+y}{a} \iff a(x + y) < b(x + y) \iff a < b$, which is given.

(iii) $\frac{x}{b} < \frac{y}{a} \iff ax < by$, which is clearly true.

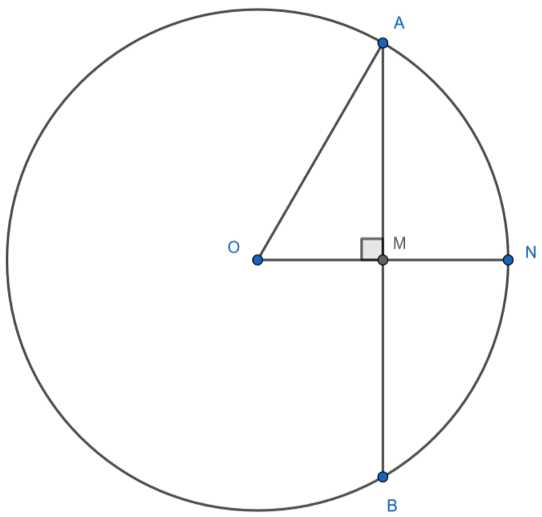
(iv) $a - y < b - x \iff a + x < b + y$, which is also clearly true.

(v) $ax < by$ is the same inequality as (iii) and is true.

7. The perpendicular bisector of a radius of a circle intersects the circle at points A and B. Given that $AB = 12$, find the radius of the circle.

(A) $2\sqrt{3}$ **(B)** 6 **(C)** $4\sqrt{3}$ **(D)** 8 **(E)** $6\sqrt{2}$

proposed by: Aidan Zhang



Solution: Let r be the length of the radius. Let O be the center of the circle and let the radius intersect AB at point M. Since $OA = OB$, side OM is shared, and $\angle OMA = \angle OMB$, $\triangle OAM \cong \triangle OBM$. Thus, $AM = BM = 6$. We have

$$OM^2 + AM^2 = OA^2 \implies \left(\frac{r}{2}\right)^2 + 6^2 = r^2.$$

Solving yields $r = \boxed{\textbf{(C) } 4\sqrt{3}}$.

8. How many possible values of a are there such that $x^2 + ax + 36$ divides the product $x(x+1)...(x+35)(x+36)$?

   (A) 3 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 12

   proposed by: Aidan Zhang

   Solution: Let the two roots of $x^2 + ax + 36$ be r and s. By Vieta's Formulas, $rs = 36$. Additionally, r and s must be two distinct values from $-1, -2, \ldots -36$ due to the divisibility condition. The possible pairs of (r, s) are thus $(-1, -36), (-2, -18), (-3, -12), (-4, -9)$. (-6,-6) does not work because there is only one factor of $(x + 6)$ in the product. Each of the pairs gives a distinct value of a, so the answer is $\boxed{\textbf{(B) } 4}$.

9. Daniel has x dollars and Elaine has y dollars. Elaine gives Daniel a **fifth** of her money, which gets added to his bank account. Daniel then gives Elaine a **third** of his total money, which gets added to her bank account. Finally, Elaine gives Daniel a **quarter** of her money, after which she notices that they both have the same amount of money. Find the ratio $\frac{x}{y}$.

   (A) $\frac{1}{2}$ **(B)** $\frac{2}{3}$ **(C)** $\frac{3}{5}$ **(D)** $\frac{4}{7}$ **(E)** $\frac{5}{7}$

   proposed by: Aidan Zhang

   Solution: Track the amount of money that Daniel and Elaine has after each transaction:

$$(x, y) \longrightarrow \left(x + \frac{y}{5}, \frac{4y}{5}\right) \longrightarrow \left(\frac{2x}{3} + \frac{2y}{15}, \frac{x}{3} + \frac{13y}{15}\right) \longrightarrow \left(\frac{3x}{4} + \frac{7y}{20}, \frac{x}{4} + \frac{13y}{20}\right)$$

   We know that at the end they have the same amount of money, so

$$\frac{3x}{4} + \frac{7y}{20} = \frac{x}{4} + \frac{13y}{20} \implies \frac{x}{y} = \boxed{\textbf{(C) } \frac{3}{5}}.$$

10. A game of whack-a-mole consists of one mole and eight holes. On each turn, the player chooses a hole to whack randomly and the mole chooses a hole to pop out of randomly. If the player plays for seventeen turns, then $\frac{m}{n}$ is the average number of times they will hit the mole, where m, n are co-prime integers. What is $m + 10n$?

    (A) 87 **(B)** 178 **(C)** 12 **(D)** 78 **(E)** 97

    proposed by: Shanna Xiao

    Solution: On each turn, the player has a $\frac{1}{8}$ chance to whack the mole. In seventeen turns, the average number of times the player will hit the mole is $17 \cdot \frac{1}{8} = \frac{17}{8}$. $m + 10n = \boxed{\textbf{(E) } 97}$.

11. A function is defined recursively with $f(0) = 1$ and $f(x) = (2x-1)f(x-1)$. Find the smallest integer n such that $f(n)$ is divisible by 2023^2.

    (A) 49 **(B)** 60 **(C)** 73 **(D)** 91 **(E)** 119

    proposed by: Aidan Zhang

Solution: First, notice that $2023^2 = 7^2 \times 17^4$. $f(x)$ has an extra factor of 17 compared to $f(x-1)$ only when $2x-1$ is a multiple of 17. In order to pick up enough factors of 17, we need to cover the 4 smallest odd multiples of 17 ($2x-1$ is always odd), which are 17, 51, 85, and 119. Thus, the smallest possible value of x is $2x-1 = 119$ or $x = \boxed{\textbf{(B) } 60}$.
Remark: We can also check that factors of 7 are gained when $x = 4$ and 11, which are smaller than 60.

12. A sequence is defined with $a_1 = 2, a_2 = 3$, and $a_n = a_{n-1}a_{n-2} - 1$ for $n > 2$. Find the last digit of a_{2023}.

    (A) 7 **(B)** 9 **(C)** 4 **(D)** 2 **(E)** 5

    proposed by: Aidan Zhang

    Solution: Notice that only the last digits of the previous terms can influence the last digit of the next term. This motivates us to write out only the last digit of the first few terms of the sequence using the recurrence relation:

    $$2, 3, 5, 4, 9, 5, 4, 9, 5 \ldots$$

    We can see that the sequence repeats every 3 terms, which means that we only need the remainder when 2023 is divided by 3 to determine its value. 2023 has remainder 1 modulo 3, so the last digit of a_{2023} is $\boxed{\textbf{(C) } 4}$.

13. Evaluate
    $$\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{9}\right)\left(1 - \frac{1}{16}\right)\ldots\left(1 - \frac{1}{2025}\right).$$

    (A) $\frac{23}{45}$ **(B)** $\frac{4}{2025}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{1013}{2025}$ **(E)** $\frac{169}{529}$

    proposed by: Charles Ran

    Solution:

    The key idea is that each term $1 - \frac{1}{n^2}$ can be expressed as $\frac{n-1}{n} \cdot \frac{n+1}{n}$. Let's use this to write out the whole expression:

    $$\left(\frac{1}{2} \cdot \frac{3}{2}\right)\left(\frac{2}{3} \cdot \frac{3}{4}\right)\cdots\left(\frac{43}{44} \cdot \frac{45}{44}\right)\left(\frac{44}{45} \cdot \frac{46}{45}\right)$$

    We can *telescope* this product by noticing that $\frac{n+1}{n}$ in each nth term cancels with the $\frac{(n+1)-1}{(n+1)}$ in the $n+1$th term. We are therefore left with simply $\frac{1}{2} \cdot \frac{46}{45} = \boxed{\textbf{(A) } \frac{23}{45}}$.

14. If $f(x)$ is a quadratic that satisfies $f(1) = f(f(1)) = 2$, compute $f(5) - 6f(3)$.

    (A) 6 **(B)** 2 **(C)** -2 **(D)** -10 **(E)** 0

    proposed by: Daniel Chen

    Solution: Notice that the roots of $f(x) - 2$ are 1 and $f(1) = 2$. We can now write

    $$f(x) - 2 = a(x-1)(x-2) \implies f(x) = ax^2 - 3ax + 2a - 2$$

    where a is the leading coefficient of the quadratic. Finally, directly plug in the desired expression:

    $$\begin{aligned} f(5) - 6f(3) &= 25a - 15a + 2a - 2 - 6(9a - 9a + 2a - 2) \\ &= \boxed{\textbf{(D) } -10}. \end{aligned}$$

15. For how many bases b, with $3 < b < 2023$, is 2023_b divisible by 7?

(A) 576 **(B)** 578 **(C)** 866 **(D)** 867 **(E)** 1156

proposed by: Aidan Zhang

Solution: Converting to base 10, $2023_b = 2b^3 + 2b + 3$. To satisfy the divisibility by 7 criteria, we must have

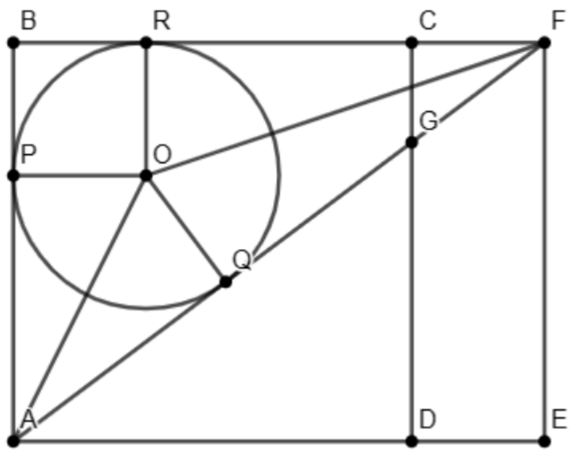
$$2b^3 + 2b + 3 \equiv 0 \mod 7.$$

Checking values of b from 0 to 6, we see that only 1 and 3 work. Thus $b \equiv 1, 3 \mod 7$, giving $2 \times 2023/7 = 578$ possible values. However, $b > 3$, which eliminates the cases 1 and 3. The answer is thus $\boxed{\textbf{(A)}\ 576}$.

16. Inside of square $ABCD$ with side length 3, a circle ω with radius 1 is drawn such that it is tangent to both $\overline{AB}$ and $\overline{BC}$. Point G lies on $\overline{CD}$ such that $\overline{AG}$ is tangent to ω. Find the area of triangle AGD.

(A) $\frac{27}{8}$ **(B)** 3 **(C)** 4 **(D)** $\frac{7}{2}$ **(E)** $\frac{15}{4}$

proposed by: Michael Li



Solution: Define points as the diagram shows below. Extend AG and BC to meet at F. Let E be on the extension of AD such that FE is perpendicular to AD. Let $GD = x$ and $CF = y$. Let P, Q, R be where ω meets AB, AG, BC, respectively.

Note that $AQ = AP = 2$ since $\triangle APO \cong \triangle AQO$ by HL congruence. Similarly, $QF = RF = 2+y$. Since $CDEF$ is a rectangle, we have $DE = CF = y \implies AE = 3+y$. By Pythagorean Theorem on $\triangle AEF$, we have:

$$AF^2 = AE^2 + FE^2 \implies (4+y)^2 = (3+y)^2 + 3^2 \implies y = 1.$$

Additionally, $\triangle GCF \sim \triangle GDA$ by AA similarity. We have:

$$\frac{CF}{CG} = \frac{AD}{DG} \implies \frac{1}{3-x} = \frac{3}{x} \implies x = \frac{9}{4}.$$

Thus, the area of $\triangle AGD$ is $\frac{1}{2} \cdot \frac{9}{4} \cdot 3 = \boxed{\textbf{(A)}\ \frac{27}{8}}$.

17. Avogadro, Bernoulli, Cauchy, Darwin, Einstein, Fermat, and Galileo are lining up to enter the exam room for OMC. How many ways are there for them to do so if Avogadro cannot be first in line, Cauchy cannot be 3rd in line, Einstein cannot be 5th in line, and Galileo cannot be last in line?

    (A) 2790 **(B)** 2250 **(C)** 6 **(D)** 2880 **(E)** 2472

    proposed by: Oscar Zhou

    Solution: Use complementary counting, and find the total number of cases minus the number of illegal cases. The number of total cases is simply $7! = 5040$. For the number of illegal cases, count by the Principle of Inclusion-Exclusion: We can directly count the cases where one of the rules are violated, but then we overcount the cases where exactly two of the rules are violated and we need to subtract them from the total. Similarly, we overcount or undercount the cases where exactly 3 or 4 of the rules are violated. Adding the 1 and 3 violation cases and subtracting the 2 and 4 violation cases eliminates overcounting. To calculate the number of cases for each number of rule violations, determine the number of ways to select who violate(s) the restrictions, and multiply by the number of ways the others can be seated. This gives a total of

    $$\binom{4}{1} \times 6! - \binom{4}{2} \times 5! + \binom{4}{3} \times 4! - \binom{4}{4} \times 3! = 2250$$

    illegal cases, so there are $5040 - 2250 = \boxed{\textbf{(A) } 2790}$ legal cases.

18. In triangle ABC, point D lies on $\overline{BC}$ such that $\angle ADB = 90°$. If $\overline{AD} = 5, \overline{BD} = 2$, and $\angle BAC = 45°$, find the length of $\overline{DC}$.

    (A) 2 **(B)** $\frac{15}{7}$ **(C)** 3 **(D)** $\frac{30}{13}$ **(E)** $\frac{30}{17}$

    proposed by: Michael Li

    Solution 1: Reflect point D across AB, AC to E, G respectively. This implies that $\triangle AEB \cong \triangle ADB$ and $\triangle AGC \cong \triangle ADC$. Thus, $\angle EAG = 90°$. Furthermore, $\angle AEB = \angle AGC = 90°$. Extend EB and GF to meet at F. This forms a square $AEFG$ with side length 5. Since $EB = 2$, $BF = 3$. Let $DC = x$. Then, $CF = GF - GC = GF - DC = 5 - x$. By Pythagorean Theorem, we have:

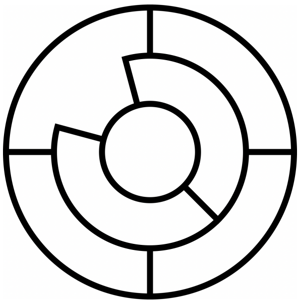
    $$BF^2 + FC^2 = BC^2 \implies 3^2 + (5-x)^2 = (2+x)^2 \implies x = \boxed{\textbf{(B) } \frac{15}{7}}.$$

    Solution 2: We will use the tangent angle subtraction formula to solve. Let $\angle BAD = \alpha$, which gives $\angle CAD = 45° - \alpha$. Then, we have:

    $$\tan(45° - \alpha) = \frac{\tan(45°) - \tan(\alpha)}{1 + \tan(45°)\tan(\alpha)} = \frac{1 - \frac{2}{5}}{1 + \frac{2}{5}} = \frac{3}{7}.$$

    Note that $\tan(45° - \alpha) = \frac{DC}{5} \implies \frac{DC}{5} = \frac{3}{7} \implies DC = \boxed{\textbf{(B) } \frac{15}{7}}$.

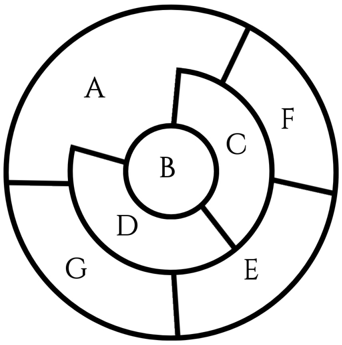
19. How many ways can the shown figure be painted with six distinct colours if no two neighboring regions can share the same colour?



(A) 9720 **(B)** 12960 **(C)** 14400 **(D)** 15480 **(E)** 17280

proposed by: Aidan Zhang

Solution: Label the regions as indicated below:



Without loss of generality, there are 6 possible choices to colour region A. Since B cannot be the same colour as A, there are 5 ways to colour B. Likewise, C borders both A and B so there are 4 choices for C, and 3 ways to colour D.

Now, we can proceed with casework on the colour of E.

Case 1: E is the same colour as A

There are now 4 possible choices to colour each of regions F and G, or $1 \times 4 \times 4 = 16$ ways in total.

Case 2: E is not the same colour as A

There are 3 possible colours for region E, since it cannot be the same colour as regions A, C, or D. Now, there are also 3 choices for both of regions F and G (since F can't be the same colour as A, C, or E, and G can't be the same colour as A, D, or E). There are thus $3 \times 3 \times 3 = 27$ ways in total in this case.

Combining the cases gives $16 + 27 = 43$ ways to colour regions E, F, G after A, B, C, D have been coloured.

The total number of valid solutions is therefore

$$6 \times 5 \times 4 \times 3 \times 43 = \boxed{\textbf{(D) } 15480}.$$

20. Player 1 and Player 2 are playing a game. On each turn, a fair coin is flipped. If it lands heads, Player 1 moves forward one square. If it lands tails, Player 2 moves forward one square. A player wins when they are 5 squares in front of the other player. If Player 1 is currently 1 square in front of Player 2, then the probability that Player 1 wins is $\frac{a}{b}$, where a, b are co-prime integers. What is $2a + 3b$?

(A) 21 **(B)** 19 **(C)** 8 **(D)** 17 **(E)** 23

proposed by: Shanna Xiao

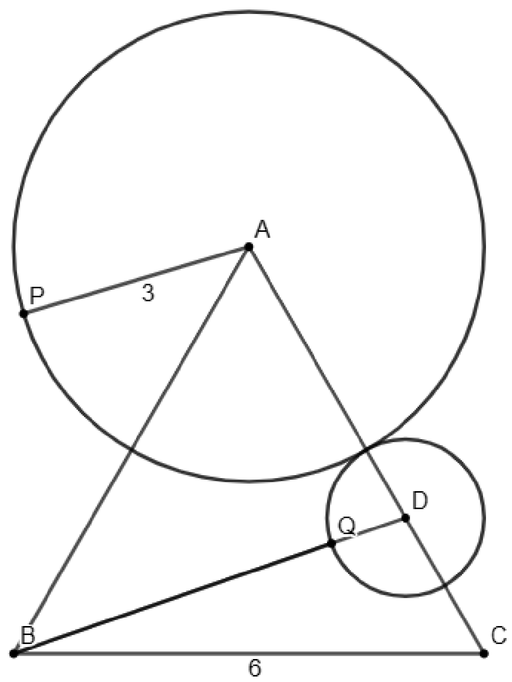
Solution 1: Let P_n be the probability Player 1 wins when it is n squares in front of Player 2. $P_0 = \frac{1}{2}$ as they are at an equal state. $P_5 = 1$ as Player 1 is at a winning state. For $n = 1$, 2, 3, 4, $P_n = \frac{1}{2}P_{n-1} + \frac{1}{2}P_{n+1}$ as there is an equal chance for player 1 to either gain one square or lose one square on the next move. Solving the system of equations, $P_1 = \frac{3}{5}$. Therefore $2a + 3b = \boxed{\textbf{(A)}\ 21}$.

Solution 2: Recognize that the problem is equivalent to a random walk starting at $(1, 0)$ and the question is asking for the probability of reaching $(5, 0)$ before $(-5, 0)$. Therefore the probability is equal to $\frac{6}{4+6} = \frac{3}{5}$, and $2a + 3b = \boxed{\textbf{(A)}\ 21}$.

21. In equilateral triangle ABC, $\overline{AB} = 6$. Consider a point P which satisfies $\overline{AP} = 3$. Let Q lie on PC such that $CQ : QP = 1 : 2$. The minimum possible length of $\overline{BQ}$ can be written in the form $a\sqrt{b} + c$, where a, b, c are integers and b is not divisible by the square of any prime. Find $a + b + c$.

(A) 11 **(B)** 12 **(C)** 8 **(D)** 10 **(E)** 14

proposed by: Michael Li



Solution: Let D be on AC such that $AD : DC = 2 : 1$. Consider the locus of points P and Q. The locus of P is a circle of radius 3 around A. The locus of Q is a circle of radius 1 around D, since we can project P to Q and A to D by scaling by a factor of $\frac{1}{3}$ about point C. Since the shortest distance from two points is a straight line, and we know that Q is on a circle around D, the shortest possible distance of BQ is $BD - QD$. By law of cosines, $BD = 2\sqrt{7}$. Thus, the shortest distance is $BQ = 2\sqrt{7} - 1$. This means that $a + b + c = 2 + 7 - 1 = \boxed{\textbf{(C)}\ 8}$.

22. The largest real solution to $x^4+2x^3-1686x^2-6x+9=0$ can be expressed as $a+\sqrt{b}$, where b is a positive integer. What is $10a+b$?

(A) 228 **(B)** 234 **(C)** 597 **(D)** 603 **(E)** 2012

proposed by: Shanna Xiao

Solution: Rearrange the equation to become $x^4+2x^3-5x^2-6x+9=1681x^2$. The left side is equal to $(x^2+x-3)^2$. Then the equation becomes $(x^2+x-3)^2=(41x)^2$. Moving all terms to the left to form a difference of squares, $(x^2+x-3+41x)(x^2+x-3-41x)=0$. This gives the two quadratics $x^2+42x-3=0$ and $x^2-40x-3=0$. Using the quadratic formula, the largest real solution to either equation is $x=20+\sqrt{403}$. Therefore $10a+b=\boxed{\textbf{(D) } 603}$.

23. How many ways are there to completely tile a 3×10 grid with 1×2 rectangles?

(A) 418 **(B)** 243 **(C)** 162 **(D)** 3888 **(E)** 571

proposed by: Jason Sun

Solution: Let T_n be the number of ways to completely tile a $3\times n$ grid with 1×2 rectangles. Notice that the number of ways to tile a $3\times n$ grid such that it cannot be decomposed into 2 smaller $3\times i$ and $3\times j$ tilings is 3 when $n=2$, and 2 otherwise. We can consider the ways to break the grid down into a sum of smaller T_i tilings, which gives us the recursion

$$T_n = 3T_{n-2}+2T_{n-4}+2T_{n-6}+\dots$$

as for each case we break the $3\times n$ grid into a $3\times 2i$ tiling which cannot be broken down further (this is to avoid overcounting) and a $3\times(n-2i)$ tiling T_{n-2i}. $T_0=1$, and we can calculate $T_2=3, T_4=11, T_6=41, T_8=153$, and finally $T_{10}=\boxed{\textbf{(E) } 571}$.

24. Parallelogram $ABCD$ has $\overline{AB}=17$ and $\overline{BC}=10$. Two circles with diameters AB and BC respectively are drawn, which intersect each other at points B and P with $\overline{BP}=8$. If the two diagonals of $ABCD$ have lengths x and y, find $|x^2-y^2|$.

(A) 266 **(B)** 538 **(C)** 104 **(D)** 310 **(E)** 907

proposed by: Aidan Zhang

Solution: Since $\overline{AB}$ and $\overline{BC}$ are diameters and P lies on their circles, $\angle APB=\angle BPC=90^\circ$. By Pythagorean Theorem on triangles $\triangle APB$ and $\triangle BPC$, $AP=15$ and $CP=6$. Thus, $AC=21$.

Let the two diagonals $\overline{AB}$ and $\overline{BC}$ intersect at point Q. Note that $\angle BPQ$ is right because Q lies on $\overrightarrow{PA}$ and $\angle BPA$ is right. Also, note that since diagonals of a parallelogram bisect each other, P is the midpoint of AC so $CQ=\frac{21}{2}$. Now, $PQ=CQ-CP=\frac{21}{2}-6=\frac{9}{2}$.

$$BQ=\sqrt{BP^2+PQ^2}=\sqrt{8^2+\left(\frac{9}{2}\right)^2}=\frac{\sqrt{337}}{2}.$$

Since Q bisects $\overline{BD}$, $BD=2BQ=\sqrt{337}$. Finally, the absolute difference between the squares of the diagonals is

$$|AC^2-BD^2|=|21^2-\sqrt{337}^2|=|441-337|=\boxed{\textbf{(C) } 104}.$$

25. The minimum value of

$$\sqrt{x^2+y^2}+\sqrt{x^2+y^2+14x+49}+\sqrt{x^2+y^2-16y+64},$$

where x and y are real numbers, is z. If $z^2 = a + b\sqrt{c}$, where c is not divisible by the square of any prime, find $a + b + c$.

(A) 200 **(B)** 128 **(C)** 256 **(D)** 131 **(E)** 172

proposed by: Michael Li

Solution: Define the points on the coordinate plane: $A(0,0), B(-7,0), C(8,0), P(x,y)$. We will turn this into a geometry problem. We are essentially finding the point P where the distance of $AP + BP + CP$ is minimized, as this is equal to the expression through the distance formula. Rotate $\triangle APC$ $60°$ outwards of $\triangle ABC$, such that P goes to P' and C goes to C'. We will prove that the shortest distance possible is BC'.

Firstly, because $\triangle AP'C'$ is a rotation of $\triangle APC$, $P'C' = PC$.
Secondly, because of the rotation, $AP = AP', \angle PAP' = 60°$. Thus, $P'P = AP$
Thirdly, $BP = BP$.

Thus, $BP, P'P, P'C'$ is a series of line segments connecting B and C'. Since the smallest distance between two points is a straight line, the minimal distance $AP + BP + CP$ is BC'. This is indeed achievable when P lies on BC'.

Note that $CC'A$ is an equilateral triangle, so C' must have coordinates $(4\sqrt{3}, 4)$. Thus, $z^2 = BC'^2 = 4^2 + (7 + 4\sqrt{3})^2 = 113 + 56\sqrt{3}$. We have $a + b + c = 113 + 56 + 3 =$ **(E)** 172.